Empirical traffic data and their implications for traffic modeling

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(Received 9 September 1996)

From single vehicle data a number of empirical results about the temporal evolution, correlation, and density dependence of macroscopic traffic quantities have been determined. These have relevant implications for traffic modeling and allow testing of existing traffic models. [S1063-651X(97)50801-4]

PACS number(s): 47.20.−k, 89.40.+k, 47.50.+d, 47.55.−t

With the aim of optimizing traffic flow and improving today's traffic situation, several models for freeway traffic have been proposed, including microscopic [1–3] and macroscopic ones [4–14]. Only some of them have been systematically derived from the underlying laws of individual vehicle dynamics [11–16]. Most models are phenomenological in nature [4–9]. These are based on various assumptions, the correctness of which has not been carefully discussed up to now, mainly due to a lack of empirical data or difficulties in obtaining them. Therefore, this paper presents some fundamental empirical observations that allow us to test some of the models.

The empirical relations have been evaluated from single vehicle data of both lanes of the Dutch freeway A9 between Haarlem and Amsterdam (cf. Fig. 1) [17]. These data were detected by induction loops at discrete places x below the lanes i of the roadway and include the passage times $t_\alpha(x,i)$, velocities $v_\alpha(x,i)$, and lengths $l_\alpha(x,i)$ of the single vehicles α. Consequently, it was possible to calculate the number $\Delta N_i(x,t)$ of vehicles on lane i which passed the cross section at place x during a time interval $[t,t+\Delta T]$, the *traffic flow*

$$\mathcal{Q}_i(x,t) := \Delta N_i(x,t)/\Delta T, \tag{1}$$

and the macroscopic *velocity moments*

$$\langle v^k \rangle_i := \frac{1}{\Delta N_i(x,t)} \sum_{t \le t_\alpha < t+\Delta T} [v_\alpha(x,i)]^k. \tag{2}$$

If nothing else is mentioned, the interval length chosen was $\Delta T = 5$ min, since this allowed us to separate the systematic temporal evolution of the macroscopic traffic quantities from their statistical fluctuations [16]. The *vehicle densities* $\rho_i(x,t)$ were calculated via the flow formula

$$\mathcal{Q}_i(x,t) = \rho_i(x,t) V_i(x,t). \tag{3}$$

A detailed comparison with other available methods for the determination of the average velocities $V_i := \langle v \rangle_i$ and the vehicle densities ρ_i from single vehicle data will be given in [16].

Finally, the *lane averages* of the above quantities were defined according to

$$\mathcal{Q}(x,t) := \frac{1}{I} \frac{\sum_i \Delta N_i(x,t)}{\Delta T} = \frac{1}{I} \sum_i \mathcal{Q}_i(x,t), \tag{4}$$

$$\langle v^k \rangle := \frac{1}{\sum_j \Delta N_j(x,t)} \sum_i \sum_{t \le t_\alpha < t+\Delta T} [v_\alpha(x,i)]^k, \tag{5}$$

and

$$\rho(x,t) := \mathcal{Q}(x,t)/V(x,t), \tag{6}$$

where I denotes the number of lanes and $V(x,t) := \langle v \rangle$. Therefore, we have the following relation:

$$\langle v^k \rangle = \sum_i \frac{\Delta N_i(x,t)}{\sum_j \Delta N_j(x,t)} \langle v^k \rangle_i . \tag{7}$$

For reasons of simplicity, most macroscopic traffic models describe the dynamics of the total cross section of the road in an overall manner by equations for the density ρ and the average velocity V. However, one would expect that a realistic description requires a model of the traffic dynamics on the single lanes and their mutual coupling due to overtaking and lane-changing maneuvers [13,16]. This could cause a more complex dynamics, such as density oscillations among the lanes [18].

In order to check this, we will investigate the correlation between neighboring lanes. Figure 2 shows that the temporal course of the densities $\rho_1(x,t)$ and $\rho_2(x,t)$ is almost parallel. The results are similar for the average velocities $V_1(x,t)$ and $V_2(x,t)$ [19]. The difference between the curves is mainly a function of density (cf. Fig. 3): At small densities, the vehicles can move faster in the left lane than in the right one,

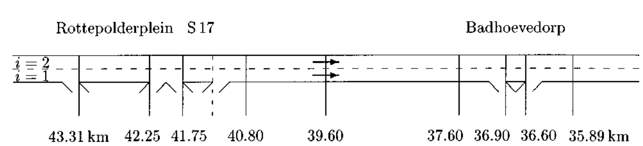

FIG. 1. The investigated stretch of the Dutch two-lane freeway A9 from Haarlem to Amsterdam including on- and off-ramps. Detectors are indicated by vertical lines. The detector at $x = 41.3$ km (---) only evaluates on-ramp traffic and a bus lane. Between $x = 40.8$ and $x = 37.6$ km traffic flow is not disturbed over more than 3 km. The speed limit is 120 km/h.

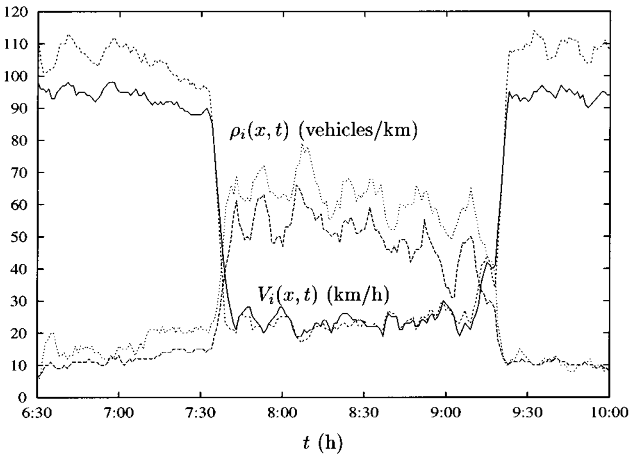

FIG. 2. The temporal course of the average velocities V_i ($-$, right lane; ---, left lane) and the vehicle densities ρ_i ($--$, right lane; $\cdots$, left lane) on October 14, 1994 at $x = 41.75$ km.

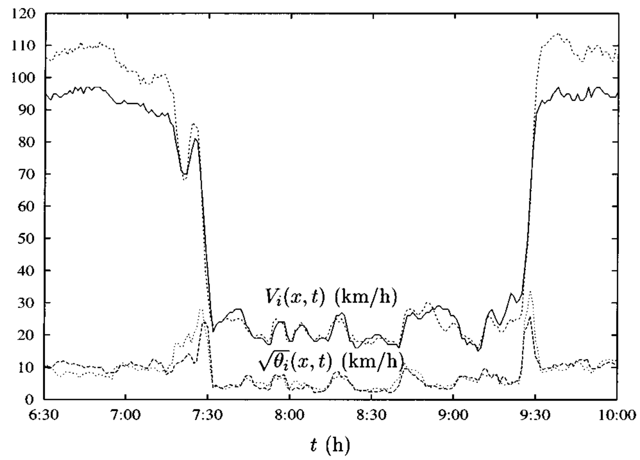

FIG. 4. The temporal course of the average velocities V_i ($-$, right lane; ---, left lane) and the standard deviations $\sqrt{\theta_i}$ of vehicle velocities ($--$, right lane; $\cdots$, left lane) on November 2, 1994 at $x = 41.75$ km.

whereas at high densities the left lane is more crowded than the right one. In addition, Fig. 4 shows that the variances

$$\theta_i(x,t) := \langle (v - V_i)^2 \rangle_i = \langle v^2 \rangle_i - (\langle v \rangle_i)^2 \tag{8}$$

behave almost identically in the neighboring lanes [although the order of magnitude of the average velocities $V_i(x,t)$ changes considerably]. This strong correlation between neighboring lanes probably arises from overtaking and lane-changing maneuvers. It justifies the common practice to describe the dynamics of the total cross section of the road in an overall manner.

Now we face the question of what a realistic traffic model must look like. Due to the conservation of the number of vehicles, the dynamics of the vehicle density is given by the *continuity equation* [4–6,13,16]

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial \mathcal{Q}(x,t)}{\partial x} = \nu^+(x,t) - \nu^-(x,t), \tag{9}$$

where $\nu^+(x,t)$ and $\nu^-(x,t)$ are the rates of vehicles which enter or leave the freeway at on- and off-ramps, respectively. Lighthill, Whitham, and Richards have suggested to specify the flow $\mathcal{Q}(x,t)$ in accordance with an empirical flow-density relation $\mathcal{Q}_e(\rho)$ [4,5]:

$$\mathcal{Q}(x,t) = \mathcal{Q}_e(\rho(x,t)). \tag{10}$$

This relation has been called into question, since the resulting model cannot describe the emergence of phantom traffic jams or stop-and-go traffic [10,16]. Therefore, some researchers have introduced an additional dynamical equation for the average velocity $V(x,t)$, which allows one to describe instabilities of traffic flow [7–10,14]. However, others have interpreted these phenomena as effects of fluctuations or of phantom bottlenecks caused by slow, overtaking vehicles like trucks [20]. Hence we check relation (10) in Fig. 5. It is found that Eq. (10) becomes invalid above a density of about

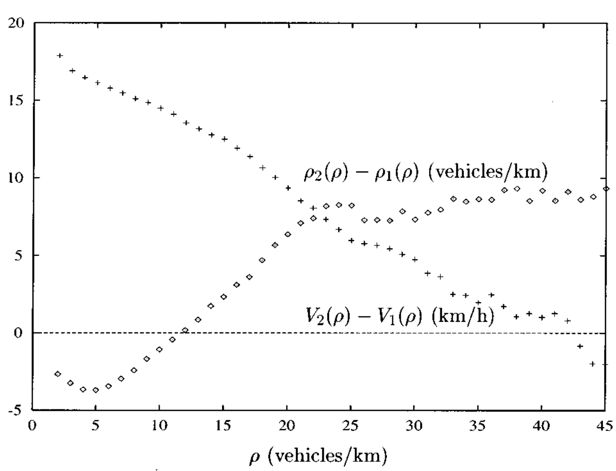

FIG. 3. Average differences between the vehicle densities ($\diamond$) and the average velocities ($+$) on both lanes of the Dutch freeway A9 on November 2, 1994 ($\Delta T = 1$ min).

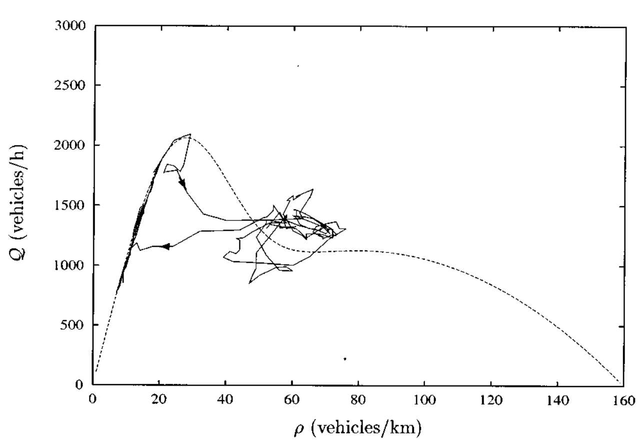

FIG. 5. Comparison of the *fundamental diagram* $\mathcal{Q}_e(\rho)$ (i.e., the average flow-density relation ($--$) with the temporal evolution of traffic flow $\mathcal{Q}(x,t) = \rho(x,t)V(x,t)$ ($-$) at $x = 41.75$ km.

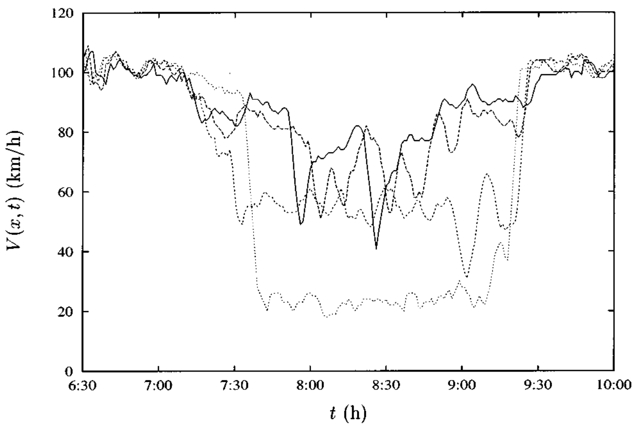

FIG. 6. Temporal evolution of the average velocity $V(x,t)$ at successive cross sections of the freeway on October 14, 1994 ($\cdots$, $x=41.75$ km; ---, $x=40.8$ km; $--$, $x=39.6$ km; —, $x=37.6$ km).

12 vehicles per kilometer and lane, where a hysteresis effect occurs [21]. This indicates a transition from stable to unstable traffic flow.

An empirical proof of emerging stop-and-go traffic is presented in Fig. 6. During the rush hours between 7:30 a.m. and 9:30 a.m. average velocity breaks down at place $x=41.75$ km because of the on-ramp at $x=41.3$ km. Nevertheless, the traffic situation recovers at the successive cross sections, i.e., average velocity increases again. In spite of this, the initially small velocity oscillations at $x=41.75$ km grow considerably in the course of the road. This corresponds to emerging stop-and-go traffic (i.e., alternating periods of acceleration and deceleration). At the same time, the wavelength of the oscillation increases. This is in good agreement with computer simulations that show a merging of density clusters leading to larger wavelengths [8].

After we have found that we need a dynamic velocity equation for an adequate description of the spatiotemporal evolution of traffic flow, we have to clear up the question of whether we also need a dynamic equation for the variance

$$\Theta := \langle (v-V)^2 \rangle = \theta + \langle (V_i - V)^2 \rangle \tag{11}$$

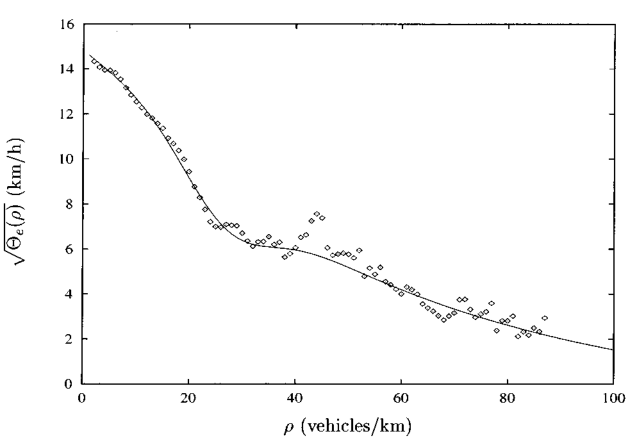

FIG. 7. Average density dependence of the standard deviation $\sqrt{\Theta_e}$ of vehicle velocities (for $\Delta T = 1$ min, $\diamond$) and corresponding fit function ($-$).

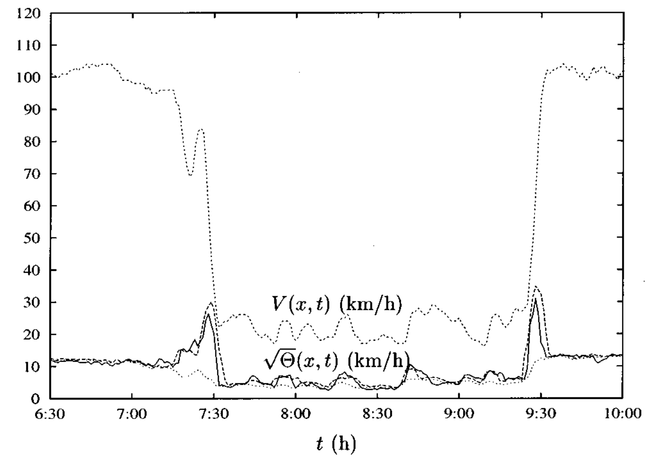

FIG. 8. Temporal evolution of the standard deviation $\sqrt{\Theta(x,t)}$ of vehicle velocities ($-$) in comparison with the equilibrium approximation $\sqrt{\Theta_e(\rho(x,t))}$ ($\cdots$). Obviously, the empirical variance $\Theta(x,t)$ has peaks where the average velocity $V(x,t)$ (---) changes considerably. In order to describe these, a correction term due to time averaging must be added ($--$). (Data from November 2, 1994 at $x=41.75$ km.)

or not. Theoretical considerations on the basis of gas-kinetic approaches have shown that the velocity equation depends on the variance, for which a separate equation can be derived [11–16]. Nevertheless we will try out the equilibrium approximation

$$\Theta(x,t) = \Theta_e(\rho(x,t)), \tag{12}$$

where $\Theta_e(\rho)$ is the empirical variance-density relation (cf. Fig. 7). Figure 8 shows that this approximation fits the temporal evolution of the variance in a satisfactory way as long as the average velocity V does not rapidly change. However, when the velocity breaks down or increases, the variance shows mysterious peaks. These are a consequence of having built the temporal averages

$$\langle v^k \rangle (x,t) \equiv \overline{\langle v^k \rangle_a}(x,t) := \frac{1}{\Delta T} \int_t^{t+\Delta T} dt' \langle v^k \rangle_a (x,t') \tag{13}$$

over finite time intervals ΔT, where $\langle v^k \rangle_a (x,t) := \int dv\, v^k P_a(v;x,t)$ with the *actual* velocity distribution $P_a(v;x,t)$. In linear Taylor approximation we find

$$V(x,t) \approx \frac{1}{\Delta T} \int_t^{t+\Delta T} dt' \left[V_a(x,t) + \frac{\partial V_a(x,t)}{\partial t} (t'-t) \right]$$
$$= V_a(x,t) + \frac{\Delta T}{2} \frac{\partial V_a(x,t)}{\partial t}. \tag{14}$$

Since $\partial V_a / \partial t$ is varying around zero, the measured value V fluctuates around the actual value $V_a(x,t) := \langle v \rangle_a (x,t)$. For the variance we find

$$\Theta \equiv \overline{\langle (v-V)^2 \rangle_a} = \overline{\Theta_a} + \overline{[V_a - V]^2}$$
$$= \overline{\Theta_a} + \frac{(\Delta T)^2}{4} \overline{\left(\frac{\partial V_a}{\partial t} \right)^2}. \quad (15)$$

Therefore, time averaging leads to a positive correction term which becomes particularly large, where the average velocity changes rapidly, but vanishes in the limit $\Delta T \rightarrow 0$. This correction term describes the variance peaks in Fig. 8 quite well. Consequently, the dynamics of the variance can be reconstructed from the dynamics of the vehicle density $\rho\ (x,t)$ and the average velocity $V(x,t)$.

Summarizing our results, we were able to demonstrate the following by empirical data: (i) The dynamics of neighboring lanes is strongly correlated so that the total freeway cross section can be described in an overall way. (ii) There is a transition from stable to unstable traffic flow at a critical densityρ_{cr} of about 12 vehicles per kilometer and lane. (iii) Emergent stop-and-go traffic exists, so that a realistic traffic model must contain a dynamic velocity equation. (iv) The variance can be well approximated by an equilibrium relation, if corrections due to time averaging are taken into account. These conclusions seem to be also valid for other stretches of freeway systems, at least European ones.

The empirical findings question the fluid-dynamic model by Lighthill, Whitham, and Richards [4,5]. They are in favor of the phenomenological models by Payne [6], Phillips [11], Kühne [7], Kerner and Konhäuser [8], and Hilliges and Weidlich [9], as well as a recent model by Helbing [14,16], which has been systematically derived from the microscopic vehicle dynamics via a gas-kinetic level of description. The last of these models fits the instability region best, in particular the surprisingly low critical density ρ_{cr} [14,16].

The author is grateful to Henk Taale and the Ministry of Transport, Public Works and Water Management for supplying the freeway data.

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